Mühendislik Matematiği / Bütünleme Sınavı / Güz 2017/Süre: 60 dakika

1) Reel kısmı $u(x,y) = y^2 - x^2 + xy$ olan bir $f(z) = u(x,y) + iv(x,y)$ (burada $z = x + iy$ dir) fonksiyonu tüm kompleks düzlemde analitiktir.a) $u(x,y)$ nin harmonik olduğunu gösterin. b) Reel kısmı verilen bu $u(x,y)$ olan ve yukarıda belirtilen $f(z)$ fonksiyonunu bulun.

2) $f(x) = -x+1$ $\quad -1 < x < 1$ *için* olarak tanımlı ve periyodu $T = 2$ olan fonksiyonun Fourier serisini elde edin.

3) a) $f(k) = 3^k u(k) + 5e^{-2k} u(k)$ darbe dizisinin Z – dönüşümünü ***serilerin toplamlarını elde ederek*** bulun. b) Bulduğunuz dönüşümün yakınsama bölgesini belirleyin. Burada $u(k)$ birim basamak fonksiyonudur.

4) $\mathfrak{F}\{ f(t) \} = \dfrac{5\omega}{6\omega^3 + 7i\omega^2 + 3}$ ise

a) $\mathfrak{F}\{ \frac{1}{3} f(\frac{t}{3}) \}$, b) $\mathfrak{F}\{ \frac{d^3}{dt^3} f(t) \}$,c) $\mathfrak{F}\{ f(t-1) \}$ d) $\mathfrak{F}\{ 4tf(t) \}$

e) $\mathfrak{F}\{ f(t)e^{-i3t} \}$ nedir?

FOURIER DÖNÜŞÜMÜ ÖZELLİKLERİ

$\mathfrak{F}\{f(t)\} = F(\omega)$ ve $\mathfrak{F}\{g(t)\} = G(\omega)$ ise;

$\mathfrak{F}\{c_1 f(t) + c_2 g(t)\} = c_1 \mathfrak{F}\{f(t)\} + c_2 \mathfrak{F}\{g(t)\} = c_1 F(\omega) + c_2 G(\omega)$

$\mathfrak{F}\{f(ct)\} = \dfrac{1}{|c|} F(\dfrac{\omega}{c}) \quad c \in R$

$\mathfrak{F}\{f(t-t_0)\} = e^{-i\omega t_0} F(\omega) \quad t_0 \in R$

$\mathfrak{F}\{e^{i\omega_0 t} f(t)\} = F(\omega - \omega_0) \quad \omega_0 \in R$

$\mathfrak{F}\{f^{(n)}(t)\} = (i\omega)^n F(\omega)$

$\mathfrak{F}\{t^n f(t)\} = i^n F^{(n)}(\omega)$

1) $u(x,y) = y^2 - x^2 + xy$

a) $\dfrac{\partial u}{\partial x} = -2x + y \qquad \dfrac{\partial^2 u}{\partial x^2} = -2$

$\dfrac{\partial u}{\partial y} = 2y + x \qquad \dfrac{\partial^2 u}{\partial y^2} = 2$

$\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = -2 + 2 = 0$

$\Rightarrow u(x,y)$ harmoniktir.

b) $\dfrac{\partial u}{\partial x} = -2x + y = \dfrac{\partial v}{\partial y} \Rightarrow v = \int \dfrac{\partial u}{\partial x} \cdot dy + g(x)$

$v = \int (-2x+y)\,dy + g(x) = -2xy + \dfrac{y^2}{2} + g(x)$

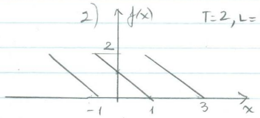
$\dfrac{\partial u}{\partial y} = 2y + x = -\dfrac{\partial v}{\partial x} = -[-2y + g'(x)]$

$\Rightarrow g'(x) = -x \Rightarrow g(x) = -\dfrac{x^2}{2} + C$

$\Rightarrow v = -2xy + \dfrac{y^2}{2} - \dfrac{x^2}{2} + C$

$f(z) = y^2 - x^2 + xy + i\left(-2xy + \dfrac{y^2}{2} - \dfrac{x^2}{2}\right) + C.$

2)



$T = 2, L = 1; \quad \dfrac{a_0}{2} = \dfrac{1}{2L} \int_{-L}^{L} f(x)\,dx = \dfrac{1}{2} \int_{-1}^{1} (-x+1)\,dx$

$\dfrac{a_0}{2} = \dfrac{1}{2} \left[-\dfrac{x^2}{2} + x \right]\Big|_{-1}^{1} = 1$

$a_n = \dfrac{1}{L} \int_{-L}^{L} f(x) \cos\left(\dfrac{n\pi x}{L}\right) dx = \dfrac{1}{1} \int_{-1}^{1} (-x+1) \cos(n\pi x)\,dx$

$= \dfrac{1}{n\pi} (-x+1) \sin n\pi x \Big|_{-1}^{1} - \int_{-1}^{1} \dfrac{1}{n\pi} (-1) \sin(n\pi x)\,dx$

$= -\dfrac{1}{(n\pi)^2} \cos(n\pi x) \Big|_{-1}^{1} = 0$

$b_n = \dfrac{1}{L} \int_{-L}^{L} f(x) \sin\left(\dfrac{n\pi x}{L}\right) dx = \dfrac{1}{1} \int_{-1}^{1} (-x+1) \sin(n\pi x)\,dx$

$= -\dfrac{(-x+1)}{n\pi} \cos(n\pi x) \Big|_{-1}^{1} + \int_{-1}^{1} \left(\dfrac{-1}{n\pi}\right) \cos(n\pi x)\,dx$

$= \dfrac{2}{n\pi} (-1)^n - \dfrac{1}{(n\pi)^2} \sin(n\pi x) \Big|_{-1}^{1} = \dfrac{2}{n\pi} (-1)^n$

3) a) $F_1(z) = \sum_{k=0}^{\infty} 3^k u(k) z^{-k} = \sum_{k=0}^{\infty} \left(\dfrac{3}{z}\right)^k = \dfrac{1}{1 - \frac{3}{z}} = \dfrac{z}{z-3}$

Y. B_1: $\left|\dfrac{3}{z}\right| < 1 \Rightarrow |z| > 3$

$F_2(z) = \sum_{k=0}^{\infty} 5e^{-2k} u(k) z^{-k}$

$F_2(z) = 5 \sum_{k=0}^{\infty} \left(\dfrac{1}{e^2 z}\right)^k = 5 \cdot \dfrac{1}{1 - \frac{1}{e^2 z}} = \dfrac{5e^2 z}{e^2 z - 1}$

Y. B_2 $\left|\dfrac{1}{e^2 z}\right| < 1 \Rightarrow |z| > e^{-2}$

$F(z) = F_1(z) + F_2(z) = \dfrac{z}{z-3} + \dfrac{5e^2 z}{e^2 z - 1}$

Y.B: Y.$B_1 \cap$ Y.B_2: $|z| > 3$

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a) $F\{ \frac{1}{3} f(\frac{t}{3}) \} = \dfrac{1}{3} \cdot \dfrac{1}{1/3} \cdot \dfrac{5 (\omega/1/3)}{6\left(\frac{\omega}{1/3}\right)^3 + 7i\left(\frac{\omega}{1/3}\right)^2 + 3} = \dfrac{5\omega}{54\omega^3 + 21i\omega^2 + 1}$

b) $F\{ \frac{d^3}{dt^3} f(t) \} = \dfrac{(i\omega)^3 5\omega}{6\omega^3 + 7i\omega^2 + 3} = \dfrac{-5i\omega^4}{6\omega^3 + 7i\omega^2 + 3}$

(c) $F\{ f(t-1) \} = e^{-i\omega} \cdot \dfrac{5\omega}{6\omega^3 + 7i\omega^2 + 3}$

(d) $F\{ 4t f(t) \} = 4 \cdot i \cdot \dfrac{d}{d\omega} F(\omega) = \dfrac{-240i\omega^3 + 140\omega^2 + 60i}{(6\omega^3 + 7i\omega^2 + 3)^2}$

(e) $F\{ f(t) e^{-i3t} \} = \dfrac{5(\omega+3)}{6(\omega+3)^3 + 7i(\omega+3)^2 + 3}$